Year 11 to 13 (ENGLISH VERSION)

Thursday, 16th March 2017 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

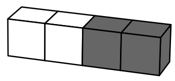
A1 $\dfrac{20 \times 17}{2+0+1+7} =$

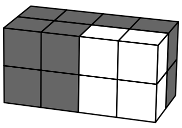
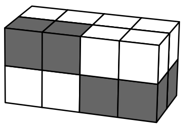
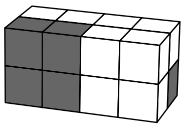
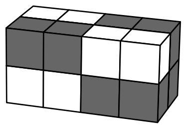
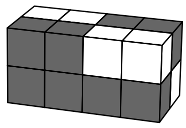
(**A**) 3.4 (**B**) 17 (**C**) 34 (**D**) 201.7 (**E**) 340

A2 Tim has an H0-model railway. He modeled some things in the H0-ratio 1 : 87, even a 2 cm high model of his brother. How tall is Tim's brother in reality?

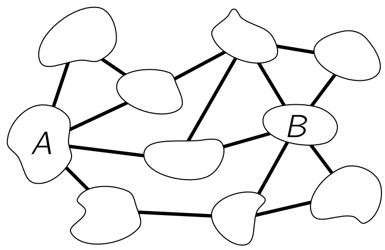
(**A**) 1.74 m (**B**) 1.62 m (**C**) 1.86 m (**D**) 1.94 m (**E**) 1.70 m

A3 Two grey cubes and two white cube were glued together to form the bar shown on the right. Which of the following cuboids could be built from four such bars?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A4 Ten islands are connected by 15 bridges, as shown in the diagram. Before a thunderstorm some of the bridges were closed temporarily. Now, there is no connection anymore from A to B. How many bridges were closed *at least*?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

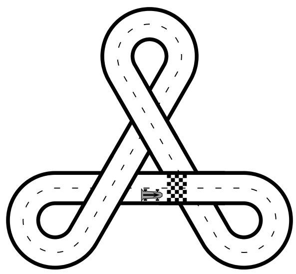
A5 The graph of the function $f(x) = x$ intersects the graphs of the following five functions. With the graph of which function does it have the *largest* number of intersection points?

(**A**) $g_1(x) = x^2$ (**B**) $g_2(x) = x^3$ (**C**) $g_3(x) = x^4$ (**D**) $g_4(x) = -x^4$ (**E**) $g_5(x) = -x$

A6 Two positive numbers a and b are such that 75 % of a equals 40 % of b. Which of the following equations is true?

(**A**) $15a = 8b$ (**B**) $7a = 8b$ (**C**) $3a = 2b$ (**D**) $5a = 12b$ (**E**) $8a = 15b$

A7 On the race track shown on the right a car completed one round. How many degrees did it rotate around its own axis?



(**A**) 360° (**B**) 540° (**C**) 720° (**D**) 900° (**E**) 1080°

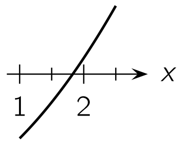
A8 Each of the following five boxes are filled with red and blue balls as labeled. Clementine wants to take one ball out of one box without looking. From which box should she take the ball to have the highest probability to get a blue ball?

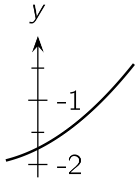
(**A**) 10 blue, 8 red (**B**) 6 blue, 4 red (**C**) 8 blue, 6 red
(**D**) 7 blue, 7 red (**E**) 12 blue, 9 red

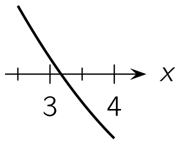
A9 The positive real number p is less than 1, and the real number q is greater than 1. Which of the following numbers is the largest?

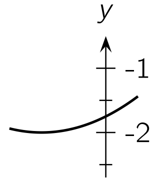
(**A**) $p \times q$ (**B**) $p + q$ (**C**) p (**D**) $\frac{p}{q}$ (**E**) q

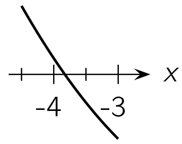
A10 Four of the following five clippings are part of the graph of the same quadratic function. Which clipping is *not* part of this graph?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

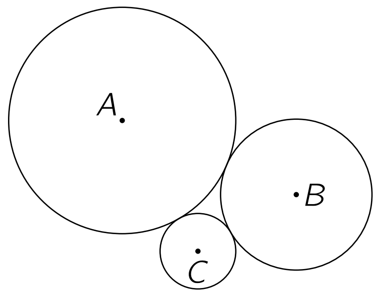
(**E**) 

4 point problems

B1 Two right cylinders have the same volume. The radius of the base of the first cylinder is by 10 % larger than that of the second cylinder. By how much is the second cylinder higher than the first cylinder?

(**A**) by 5 % (**B**) by 9 % (**C**) by 11 % (**D**) by 20 % (**E**) by 21 %

B2 Three mutually tangent circles with centres A, B, C have the radii 3 cm, 2 cm and 1 cm, respectively. What is the area of the triangle ABC ?

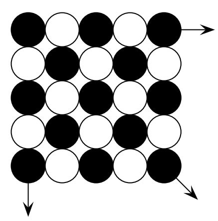


(**A**) $6\,\text{cm}^2$ (**B**) $4\sqrt{3}\,\text{cm}^2$ (**C**) $3\sqrt{2}\,\text{cm}^2$
(**D**) $9\,\text{cm}^2$ (**E**) $2\sqrt{6}\,\text{cm}^2$

B3 Andrew wants to cut a piece of string into 9 pieces of equal length. He marks his cutting points on the string. Rachel wants to cut the same piece of string into 15 pieces of equal length. She also marks her cutting points on the string. Mary then cuts the string at all the cutting points that are marked. How many pieces of string does Mary obtain?

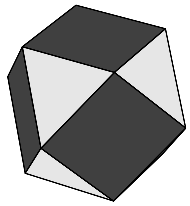
(**A**) 21 (**B**) 22 (**C**) 23 (**D**) 24 (**E**) 25

B4 Julia has 109 black chips and 108 white chips. She places them in a square pattern as shown, starting with a black chip in the upper left corner, alternating colours in each row and each column. How many chips of each colour are left after Julia has completed the largest possible such square?



(**A**) 12 black and 10 white (**B**) 11 black and 10 white
(**C**) 11 black and 11 white (**D**) 10 black and 11 white (**E**) 10 black and 10 white

B5 The faces of the solid shown are either equilateral triangles or squares. Each square is surrounded by 4 triangles, and each triangle is surrounded by 3 squares. There are 6 squares in total. How many triangles are there in total?

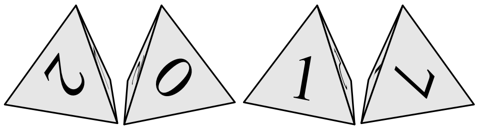


(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

B6 Jarkko and Ville are about to take a plane to Finland. Their luggage weighs 60 kg in total. At the airport they are told that they exceed the maximal allowed weight for free luggage. For the excess luggage they have to pay a fixed amount of money per kilogram, in total 112 Euros. Jarkkos says: "If I flew alone with these 60 kg, I would even have to pay 296 Euros!". What is the allowed weight for free luggage on this flight?

(**A**) 18 kg (**B**) 19.5 kg (**C**) 20 kg (**D**) 23 kg (**E**) 25.5 kg

B7 We have four tetrahedrons, perfectly balanced, with their faces numbered 2, 0, 1 and 7. If we roll all four tetrahedrons, what is the probability that we can compose the number 2017 using exactly one of the three visible numbers from each tetrahedron, as shown in the example on the right?

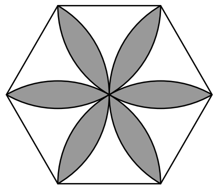


(**A**) $\frac{31}{32}$ (**B**) $\frac{63}{64}$ (**C**) $\frac{81}{256}$ (**D**) $\frac{29}{32}$ (**E**) $\frac{13}{16}$

B8 Two consecutive positive integers M and $M+1$ are such that the sum of the digits of each of them is a multiple of 7. *At least* how many digits does M have?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

B9 The diagram shows a regular hexagon with side length 1. The grey flower was constructed with sectors of circles with radius 1 and centers in the vertices of the hexagon. What is the area of the grey flower?



(**A**) $\frac{\pi}{2}$ (**B**) $\frac{2\pi}{3}$ (**C**) $2\sqrt{3}-\pi$ (**D**) $\frac{\pi}{2}+\sqrt{3}$ (**E**) $2\pi-3\sqrt{3}$

B10 Exactly four of the following six statements about the 2-digit integer N are true:

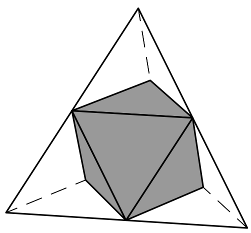
(1) $N < 30$ (3) One of the digits of N is 2. (5) N is divisible by 3.
(2) $N > 50$ (4) One of the digits of N is 6. (6) N is divisible by 5.

What is the sum of the digits of the number N?

(**A**) 3 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) 13

5 point problems

C1 The four corners of a regular tetrahedron are cut off by four planes, each passing through the midpoints of three adjacent edges, as shown. What fraction of the volume of the original tetrahedron is the volume of the resulting solid, which is marked in grey?



(**A**) $\frac{4}{5}$ (**B**) $\frac{3}{4}$ (**C**) $\frac{2}{3}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{1}{3}$

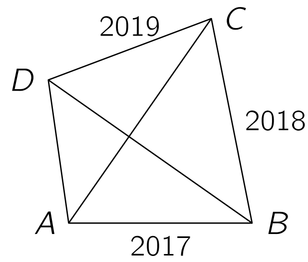
C2 The polynomial $5x^3+ax^2+bx+24$ has integer coefficients a and b. Which of the following is *certainly not* a root of this polynomial?

(**A**) $x=1$ (**B**) $x=-1$ (**C**) $x=3$ (**D**) $x=5$ (**E**) $x=6$

C3 A sequence of numbers a_n is given by $a_1=2017$ and $a_{n+1}=\dfrac{a_n-1}{a_n}$ for all integers $n\geq 1$. What is a_{2017}?

(**A**) -2017 (**B**) $-\dfrac{1}{2016}$ (**C**) $\dfrac{2016}{2017}$ (**D**) $\dfrac{2017}{2016}$ (**E**) 2017

C4 In the quadrilateral $ABCD$ the diagonals intersect in the interior and are perpendicular. $|AB|=2017$, $|BC|=2018$ and $|CD|=2019$. What is $|AD|$? (*diagram not to scale*)



(**A**) 2016 (**B**) 2018 (**C**) $\sqrt{2020^2-4}$
(**D**) 2020 (**E**) $\sqrt{2018^2+2}$

C5 How many three-digit positive integers $\overline{abc}$ exist, such that $(a+b)^c$ is a three-digit integer and an integer power of 2?

(**A**) 16 (**B**) 18 (**C**) 19 (**D**) 21 (**E**) 24

C6 The sum of the lengths of the three sides of a right-angled triangle is 18 cm and the sum of the squares of the lengths of the three sides is $128\,\text{cm}^2$. What is the area of this triangle?

(**A**) $18\,\text{cm}^2$ (**B**) $16\,\text{cm}^2$ (**C**) $12\,\text{cm}^2$ (**D**) $10\,\text{cm}^2$ (**E**) $9\,\text{cm}^2$

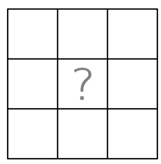
C7 Each of the 513 people living on an island is either a knight who always tells the truth or a knave who always lies. When more than 400 of the islanders took part in a banquet, they were all sitting around a huge round table. Each of them claimed: "Of the two people beside me, one is a knight and the other one a knave." How many knights are there on the island *at most*?

(**A**) 312 (**B**) 313 (**C**) 342 (**D**) 379 (**E**) 401

C8 If two real numbers x and y fulfill $|x|+x+y=5$ and $x+|y|-y=10$, what is the value of $x+y$?

(**A**) 1 (**B**) -5 (**C**) 3 (**D**) 25 (**E**) -15

C9 In each cell of a 3×3 table an integer is written. The sum of all 9 integers is 777. Any two numbers in cells that share a common side differ by 1. Which number could be in the central cell?



(**A**) only 85 (**B**) only 85 and 86 (**C**) only 85 and 87
(**D**) only 85, 86 and 87 (**E**) only 87

C10 During their holidays Jonas, Leo and Tobias played badminton every day. They agreed that the winner of each match continues to play in the next match against the one who just had a break, while the loser of this match pauses. At the end of their holidays, it turned out that Jonas played 18 matches, and Leo played 25 matches. What is the *largest* number of matches that Tobias could have played?

(**A**) 29 (**B**) 31 (**C**) 33 (**D**) 35 (**E**) 39