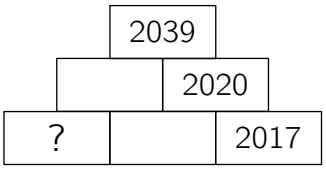
Year 9 and 10 (ENGLISH VERSION)

Thursday, 16th March 2017 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

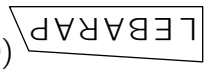
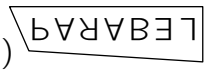
3 point problems

A1 In the number pyramid shown each number above the bottom row is the sum of the two numbers immediately below it. Which number should appear in the cell with the question mark?



(**A**) 15 (**B**) 16 (**C**) 17 (**D**) 18 (**E**) 19

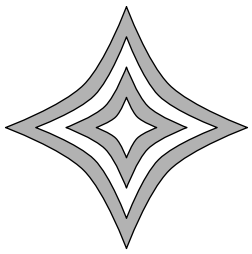
A2 Franzi wrote the German word PARABEL on a piece of transparent foil, as shown. What will she see if she turns this piece over downwards?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A3 Some girls were dancing in a circle. Between Antonia and Bianca there were 8 girls in one direction and 11 girls in the other direction. How many girls were in the group?

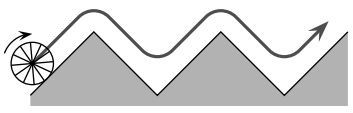
(**A**) 19 (**B**) 20 (**C**) 21 (**D**) 22 (**E**) 23

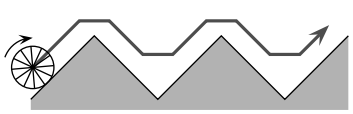
A4 The diagram on the right shows four overlapping stars, two light ones and two dark ones. The areas of the stars are $64\,\text{cm}^2$, $36\,\text{cm}^2$, $16\,\text{cm}^2$ and $4\,\text{cm}^2$. What is the total dark area?



(**A**) $36\,\text{cm}^2$ (**B**) $40\,\text{cm}^2$ (**C**) $45\,\text{cm}^2$ (**D**) $46\,\text{cm}^2$ (**E**) $49\,\text{cm}^2$

A5 Which of the following diagrams shows the locus of the midpoint of the wheel when the wheel rolls along the zig-zag curve shown?

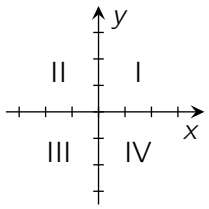
(**A**)  (**B**)  (**C**) 

(**D**)  (**E**) 

A6 Olivia has 25 euros. Each of her three sisters has 9 euros. How many euros does Olivia need to give to each of her sisters so that each of the four girls has the same amount of money?

(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

A7 Which quadrant of the coordinate system contains *no* points of the straight line with the equation $y = -1.5x + 2$?

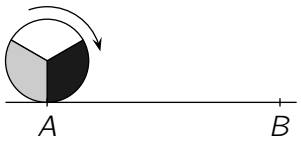


(**A**) I (**B**) II (**C**) III (**D**) IV (**E**) All quadrants are passed.

A8 Martina plays chess. She has played 15 games this season, out of which she has won 9. She has 5 more games to play. What will her success rate be in this season if she wins all 5 remaining games?

(**A**) 60 % (**B**) 65 % (**C**) 70 % (**D**) 75 % (**E**) 80 %

A9 A circle of radius 1 rolls along a straight line from the point A to the point B, where $|AB| = 11\pi$. Which of the following pictures shows the correct appearance of the circle when it reaches B?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A10 One-eighth of the guests at a wedding were children. Three-sevenths of the adult guests were women. What fraction of the wedding guests were adult men?

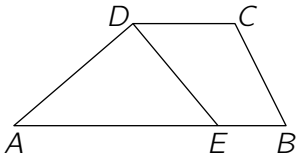
(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{2}{5}$ (**D**) $\frac{3}{7}$ (**E**) $\frac{4}{7}$

4 point problems

B1 I have a box that contains buttons of three different colours, white, blue and green. There are at least 3 of each colour. My sister is blindfolded and takes some buttons from the box at random. How many buttons does she need to take *to be sure* that she has taken at least 3 buttons of the same colour?

(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 7 (**E**) 9

B2 As shown in the diagram (*which is not to scale*), $ABCD$ is a trapezium with side $\overline{AB}$ parallel to side $\overline{CD}$. The point E is on the side $\overline{AB}$ such that the segment $\overline{DE}$ divides the trapezium into two parts of equal area. The lengths of $\overline{AB}$ and $\overline{CD}$ are 50 cm and 20 cm, respectively. What is the length of $\overline{AE}$?



(**A**) 25 cm (**B**) 30 cm (**C**) 35 cm (**D**) 40 cm (**E**) 45 cm

B3 Four brothers have different heights. Tobias is shorter than Victor by the same amount by which he is taller than Peter. Oscar is shorter than Peter by the same amount, as well. Tobias is 184 cm tall, and the average height of all the four brothers is 178 cm. How tall is Oscar?

(**A**) 160 cm (**B**) 164 cm (**C**) 166 cm (**D**) 170 cm (**E**) 172 cm

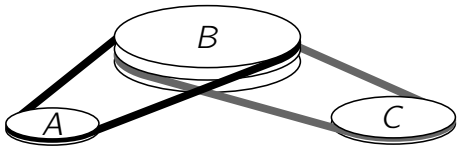
B4 If $2x + 2y = 14$ and $x^2 - y^2 = 21$, what is $x - y$?

(**A**) 1 (**B**) -3 (**C**) 2 (**D**) 3 (**E**) -1

B5 Merle wants to prepare a jogging timetable for the next few months. She wants to jog exactly three days per week and on the same days every week. She never wants to jog on two consecutive days. How many different timetables could Merle prepare?

(**A**) 6 (**B**) 7 (**C**) 10 (**D**) 12 (**E**) 13

B6 A belt drive system consists of the wheels A, B and C, which rotate without any slippage. The wheel B makes 4 full turns when A makes 5 full turns; also B makes 6 full turns when C makes 7 full turns. The perimeter of wheel C is 30 cm. What is the perimeter of wheel A?



(**A**) 27 cm (**B**) 28 cm (**C**) 29 cm (**D**) 30 cm (**E**) 31 cm

B7 How many positive integers N possess the property that either the number N or the number $N + 20$ is a 3-digit number?

(**A**) 19 (**B**) 20 (**C**) 38 (**D**) 39 (**E**) 40

B8 Maja wants to write a number into each cell of a 3×3 grid. She wants to do this in such a way that the numbers in each of the four 2×2 grids that form part of the 3×3 grid have the same totals. Maja has already written numbers in three of the corner cells, as shown. Which number does she need to write in the bottom right corner?

3		1
2		?

(**A**) 6 (**B**) 5 (**C**) 4 (**D**) 1 (**E**) 0

B9 The sum of the squares of three consecutive positive integers is 770. What is the largest of these integers?

(**A**) 15 (**B**) 16 (**C**) 17 (**D**) 18 (**E**) 19

B10 Ivana has two identical dice. On the six faces of each are the numbers −3, −2, −1, 0, 1 and 2. If Ivana throws her dice and multiplies the results, what is the probability that their product is negative?

(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{13}{36}$ (**E**) $\frac{1}{3}$

5 point problems

C1 For an integer z exactly two of the following five inequalities are true, while the other three are false.

(1) $2z > 130$ (2) $z < 200$ (3) $3z > 50$ (4) $z > 205$ (5) $z > 15$

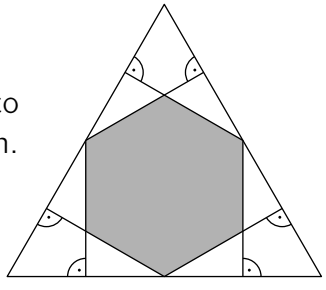
Which are the two inequalities that are true?

(**A**) (1) and (3) (**B**) (3) and (4) (**C**) (2) and (3) (**D**) (4) and (5) (**E**) (2) and (5)

C2 The product of four different positive integers, all of which are less than 20, is 882. What is the sum of these four positive integers?

(**A**) 23 (**B**) 25 (**C**) 27 (**D**) 31 (**E**) 33

C3 From the midpoint of each side of an equilateral triangle the perpendiculars to the other two sides are drawn. These perpendiculars form a hexagon, as shown. What fraction of the area of the initial triangle does this hexagon cover?

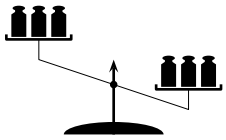


(**A**) $\frac{1}{3}$ (**B**) $\frac{2}{5}$ (**C**) $\frac{4}{9}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{5}{9}$

C4 For the warm-up in the drama club 18 actors are standing in a circle and facing the centre. At a signal some dancers randomly turn to the left, and all the others turn to the right. Now, 8 actors are facing another actor and say "Hello" to him. At a second signal all the actors make a half-turn. Again, those actors who are facing another actor say "Hello". How many actors say "Hello" this time?

(**A**) 4 (**B**) 8 (**C**) 10
(**D**) 16 (**E**) This is not uniquely determined.

C5 On a balance scale, three different masses were put at random on each pan, and the result is shown in the picture. The masses are of 101, 102, 103, 104, 105 and 106 grams. What is the probability that the 106 gram mass stands on the heavier pan?

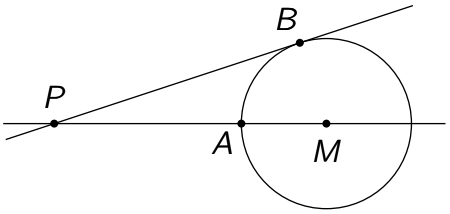


(**A**) 40 % (**B**) 60 % (**C**) 80 % (**D**) 90 % (**E**) 100 %

C6 Liza wanted to find the total of the interior angles of an n-gon, which has interior angles with different measures all of which are less than 180°. However, Liza missed one of the angles and obtained the result 2017°. What is n?

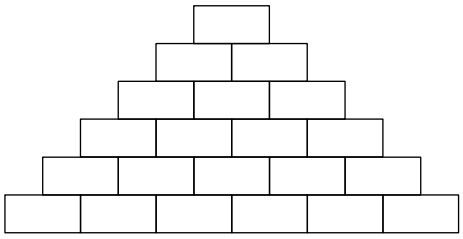
(**A**) 12 (**B**) 13 (**C**) 14 (**D**) 16 (**E**) 18

C7 The points A and B are on the circle with centre M, and PB is tangent to the circle at B. The point A lies on the straight line through P and M. The distances $|PA|$ and $|MB|$ are integers, and $|PB| = |PA| + 6$. How many possible values are there for $|MB|$?



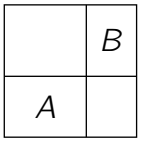
(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 8

C8 Carlos wants to put numbers in the number pyramid shown in such a way that each number above the bottom row is the sum of the two numbers immediately below it. What is the *largest* number of *odd* numbers that Carlos could put in the pyramid?



(**A**) 13 (**B**) 14 (**C**) 15 (**D**) 16 (**E**) 17

C9 A square with side length 33 m is divided into four rectangles, as shown. Rectangle A has area 90 m^2, and rectangle B has circumference 90 m. What is the area of the smaller one of the other two rectangles? (*diagram not to scale*)



(**A**) 108 m^2 (**B**) 240 m^2 (**C**) 207 m^2 (**D**) 75 m^2 (**E**) 186 m^2

C10 During their holidays Jonas, Leo and Tobias played badminton every day. They agreed that the winner of each match continues to play in the next match against the one who just had a break, while the loser of this match pauses. At the end of their holidays, it turned out that Jonas played 17 matches, and Leo played 23 matches. What is the *smallest* number of matches that Tobias could have played?

(**A**) 12 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 20