Year 11 to 13 (ENGLISH VERSION)

Thursday, 21th March 2019 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

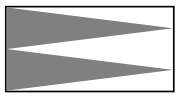
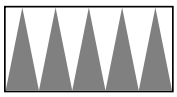
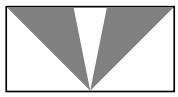
A1 The flag shown consists of three rectangular pieces of fabric of the same dimensions. The flag is 50 cm high. What is the width of the flag?

(**A**) 50 cm (**B**) 75 cm (**C**) 100 cm (**D**) 125 cm (**E**) 150 cm

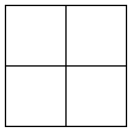
A2 A newborn panda is tiny and weighs only about 100 g. That is about $\frac{1}{900}$ of its mother's weight. How much does its mother weigh?

(**A**) about 11 kg (**B**) about 90 kg (**C**) about 111 kg (**D**) about 900 kg (**E**) about 1111 kg

A3 A rectangle has been shaded in five different ways. In which picture is the area of the shaded part the largest?

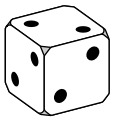
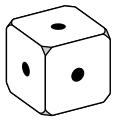
(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A4 The numbers 1, 2, 3 and 4 are written into the four empty cells on the right. After that, the sum of the numbers in each row and each column is calculated. Two of these sums are 4 and 5. What are the other two sums?



(**A**) 6 and 6 (**B**) 3 and 5 (**C**) 4 and 5 (**D**) 4 and 6 (**E**) 5 and 6

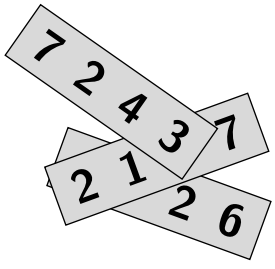
A5 Each of the faces of Kira's lucky die is marked with either 1, 2 or 3 dots. The probability of rolling a 1 is $\frac{1}{2}$, the probability of rolling a 2 is $\frac{1}{3}$ and the probability of rolling a 3 is $\frac{1}{6}$. Only one of the following dice can be Kira's lucky die. Which one?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A6 A pyramid has 23 triangular faces. How many edges does this pyramid have?

(**A**) 23 (**B**) 24 (**C**) 46 (**D**) 48 (**E**) 69

A7 Three 4-digit numbers are written on three pieces of paper as shown. The sum of the three numbers is 11126. What are the three covered digits?



(**A**) 1, 4, 7 (**B**) 1, 5, 7 (**C**) 3, 3, 3 (**D**) 4, 5, 6 (**E**) 4, 5, 7

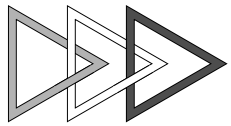
A8 At Antonia's birthday party she sits with her 5 friends at a round table to play a card game. Antonia sits directly between Maats and Luis, Zorah sits directly to Gustav's right, and Denise does not sit directly opposite Luis. What is true?

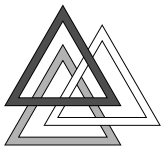
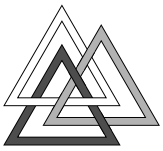
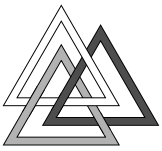
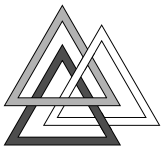
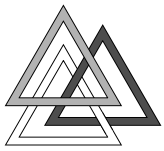
(**A**) Denise sits next to Luis. (**B**) Maats sits next to Denise. (**C**) Gustav sits next to Luis.
(**D**) Maats sits next to Luis. (**E**) Luis sits next to Zorah.

A9 What is the first (leftmost) digit of the smallest natural number whose digits add up to 42?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

A10 Three triangles are linked as shown in the diagram on the right. Which of the following diagrams shows these three triangles linked in the same way?



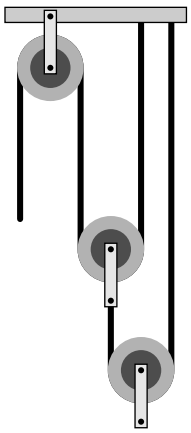
(**A**) 
(**B**) 
(**C**) 
(**D**) 
(**E**) 

4 point problems

B1 Yanis, Max and Fedor meet every day to play frisbee at the lake. They have noticed the following: Whenever Yanis doesn't bring a frisbee, Max brings a frisbee. Whenever Max doesn't bring a frisbee, Fedor brings a frisbee. Yesterday Max didn't bring a frisbee. Who brought a frisbee yesterday?

(**A**) only Yanis (**B**) only Fedor (**C**) Yanis and Fedor
(**D**) none of the three (**E**) This cannot be said for sure.

B2 The pully shown has three wheels. The visible parts of the rope run perpendicular to the ceiling. How much does the lowest wheel move upwards when the lefthand end of the rope is pulled down 24 cm?

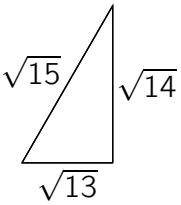


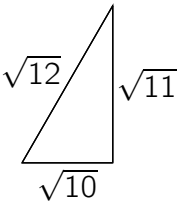
(**A**) 12 cm (**B**) 9 cm (**C**) 8 cm (**D**) 6 cm (**E**) 4 cm

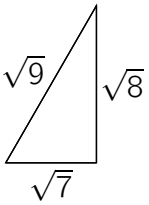
B3 This school year there are 20 % more girls and 20 % fewer boys in Laura's computer science course than in the last school year. All in all, the number of students has increased by 1 compared to the last school year. How many students in total could be in Laura's computer science course this school year?

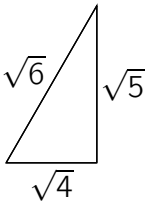
(**A**) 16 (**B**) 18 (**C**) 21 (**D**) 24 (**E**) 27

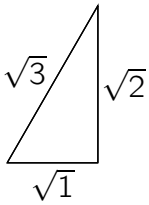
B4 The following five triangles with their side lengths are sketched so that it looks as if they were all right-angled. In fact, only one of them is right-angled. Which one?

(**A**) 

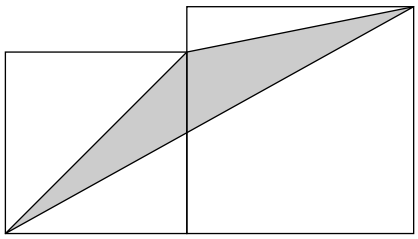
(**B**) 

(**C**) 

(**D**) 

(**E**) 

B5 In the picture on the right, the left square has side length a and the right square has side length b. What is the area of the grey triangle?

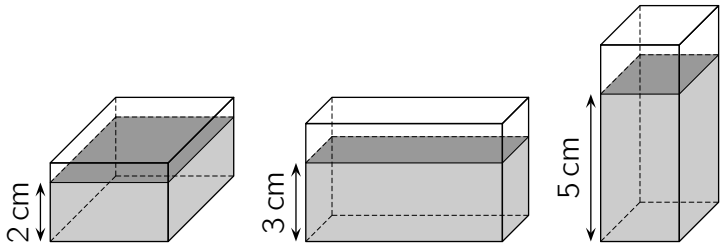


(**A**) $b^2 - a^2$ (**B**) $\frac{a^2}{2}$ (**C**) $\frac{a^2+b^2}{4}$ (**D**) $4(b-a)^2$ (**E**) $\frac{ab}{2}$

B6 How many of the natural numbers from 2^{10} to 2^{13}, both numbers included, are divisible by 2^{10}?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 16

B7 A cuboid glass box contains 120 cm^3 of water. Depending on which side the box stands on, the water-level is 2 cm, 3 cm or 5 cm (*diagram not to scale*). What is the volume of the glass box?



(**A**) 160 cm^3 (**B**) 180 cm^3 (**C**) 200 cm^3 (**D**) 220 cm^3 (**E**) 240 cm^3

B8 Heidi and Achim share the last 5 chocolates that are left over from Christmas. The chocolates look completely the same, exactly one is filled with nougat. Heidi is curious: "Let's see if I can get the nougat chocolate." Heidi and Achim take turns picking one chocolate from the box. Achim begins. What is the probability that Heidi will get the nougat chocolate?

(**A**) $\frac{2}{5}$ (**B**) $\frac{3}{5}$ (**C**) $\frac{1}{2}$ (**D**) $\frac{5}{6}$ (**E**) $\frac{1}{3}$

B9 Which of the following statements is true for $w = \sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20}}}}}$?

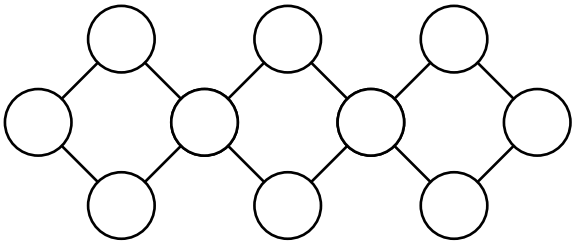
(**A**) $4 < w < 5$ (**B**) $5 < w < 6$ (**C**) $6 < w < 7$ (**D**) $20 < w < 21$ (**E**) $25 < w < 26$

B10 On a circular disk that is rotating around its center, two points P and Q are marked. P is 3 cm farther from the center than Q. The speed at which P moves is constant and two and a half times the speed at which Q moves. How far is P from the center of the disc?

(**A**) 10 cm (**B**) 9 cm (**C**) 8 cm (**D**) 6 cm (**E**) 5 cm

5 point problems

C1 The numbers from 1 to 10 should be written into the 10 circles in the picture, such that in each of the three squares, the sum of the numbers in the four corners is the same. What is the smallest possible value of this sum?



(**A**) 18 (**B**) 19 (**C**) 20 (**D**) 21 (**E**) 22

C2 The sequence $a_1, a_2, a_3, \ldots$ starts with $a_1 = 49$, and the other elements of the sequence are calculated as follows: We get a_{n+1} by taking the sum of digits of a_n, adding 1 and then squaring the result. So $a_2 = (4+9+1)^2 = 14^2 = 196$, $a_3 = (1+9+6+1)^2 = 17^2 = 289$, and so on. What is a_{2019}?

(**A**) 121 (**B**) 25 (**C**) 64 (**D**) 400 (**E**) 49

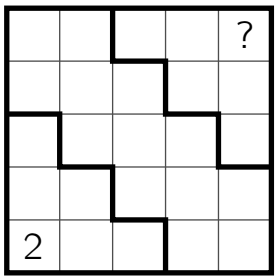
C3 A straight line through the coordinate origin intersects the graph of the function $f(x) = x^2 - 2$ in two points. What could be the product of the x coordinates of these two points?

(**A**) only -2
(**B**) only 2
(**C**) any value less than or equal to -2
(**D**) any value greater than or equal to 2
(**E**) any value greater than or equal to -2

C4 From the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ we randomly select three different numbers. What is the probability that one of the selected numbers will be the arithmetic mean of the other two?

(**A**) $\frac{1}{10}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{1}{2}$

C5 The numbers 1, 2, 3, 4 and 5 are to be written in the cells of the 5×5 field, such that in each row and in each column each of these five numbers occurs exactly once. Furthermore, the sums of the numbers in the three thickly black-rimmed parts must be equal. Which number belongs in the cell with the question mark?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

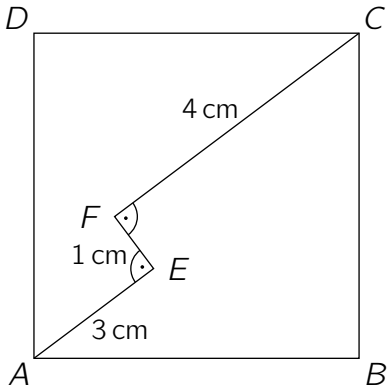
C6 How many planes are there that contain at least three corners of a given cube?

(**A**) 6 (**B**) 12 (**C**) 16 (**D**) 20 (**E**) 24

C7 For how many integers k is $\left|k^2 - 2k - 3\right|$ a prime number?

(**A**) 1 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 7

C8 Inside the square $ABCD$ the points E and F are located such that $\measuredangle FEA = \measuredangle EFC = 90°$, $|AE| = 3\,\text{cm}$, $|EF| = 1\,\text{cm}$ and $|FC| = 4\,\text{cm}$ (*diagram not to scale*). What is the side length of the square $ABCD$?



(**A**) $3\sqrt{2}\,\text{cm}$ (**B**) $5.5\,\text{cm}$ (**C**) $\frac{7\sqrt{2}}{2}\,\text{cm}$ (**D**) $5\,\text{cm}$ (**E**) $4\sqrt{2}\,\text{cm}$

C9 We call n the sum of all positive divisors of 1024. What is the product of all these divisors?

(**A**) $(n-1)^5$ (**B**) $(n+1)^5$ (**C**) n^5 (**D**) $n^5 - 1$ (**E**) $n^5 + 1$

C10 The equation $2 - |x| = ax$ with the real parameter a has two real solutions if and only if the parameter a satisfies

(**A**) $a < -1$ (**B**) $-1 \leq a \leq 1$ (**C**) $a \geq 1$ (**D**) $a = 0$ (**E**) $-1 < a < 1$