Year 9 and 10 (ENGLISH VERSION)

Thursday, 21st March 2019 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

A1 A model railway needs exactly 1 min 11 s for each round. How long does it need for 6 rounds?

(**A**) 6 min 56 s (**B**) 7 min 6 s (**C**) 7 min 16 s (**D**) 7 min 26 s (**E**) 7 min 36 s

A2 While the hairdresser is cutting my hair, the apprentice has to write HAIRCUT on the wall behind me so that I can read it correctly in the mirror in front of me. What does he have to write?

(**A**) HAIRCUT (**B**) TUCRIAH (**C**) HAIRƆUT
(**D**) TUƆRIAH (**E**) TUƆRIAH

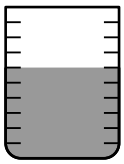
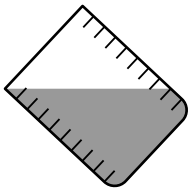
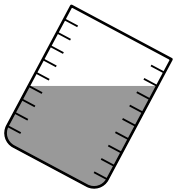
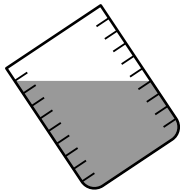
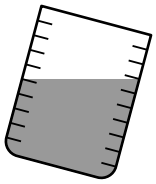
A3 Three standard dice are rolled at the same time. How many different sums of points are possible?

(**A**) 13 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 21

A4 $\dfrac{20-19\times 20+19}{19-20\times 19+20}=$

(**A**) -419 (**B**) -39 (**C**) 1 (**D**) 39 (**E**) 381

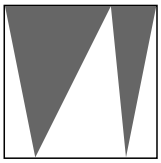
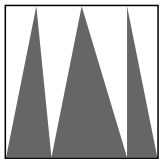
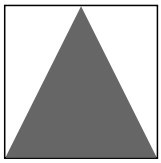
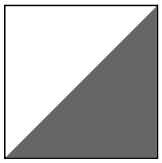
A5 Five identical glasses are filled with water and tilted differently. Four of the glasses contain the same amount of water. In which glass is the amount of water different?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

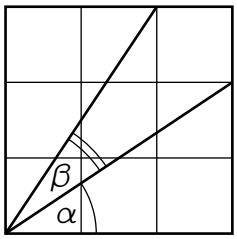
A6 In the wall around our city park there are 5 gates. How many possibilities are there to enter the park through one of the gates and exit through another?

(**A**) 24 (**B**) 20 (**C**) 16 (**D**) 15 (**E**) 12

A7 A square has been shaded in five different ways. In which picture is the area of the shaded part the largest?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A8 A square is divided into nine equal squares as shown in the diagram. Which of the following equations for the angles α and β is correct?

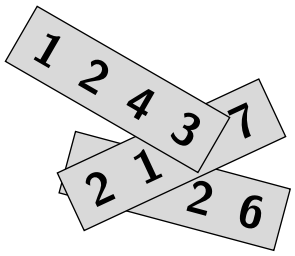


(**A**) $\alpha+\beta=45°$ (**B**) $2\beta+\alpha=90°$ (**C**) $\alpha+\beta=60°$
(**D**) $2\alpha+\beta=90°$ (**E**) $\alpha=\beta$

A9 During packing the prizes for the Kangaroo math competition, Martin put three parcels together on the scales. Their total weight is 28 kg. The weights of the three parcels, given in kilograms, are three different natural numbers. How much can the lightest of these parcels weigh *at most*?

(**A**) 1 kg (**B**) 4 kg (**C**) 7 kg (**D**) 8 kg (**E**) 9 kg

A10 There is a 4-digit number on each of the three pieces of paper shown. I know that their sum is 10126. What are the three hidden digits?



(**A**) 5, 6, 7 (**B**) 4, 5, 7 (**C**) 4, 6, 7 (**D**) 4, 5, 6 (**E**) 5, 7, 8

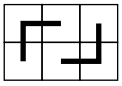
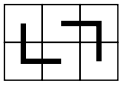
4 point problems

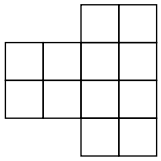
B1 A square $ABCD$ and an equilateral triangle ACE have been constructed in such a way that the point D lies inside the equilateral triangle. What is the size of the angle EDA?

(**A**) 30° (**B**) 72° (**C**) 135° (**D**) 144° (**E**) 150°

B2 The numbers a, b, c and d are distinct numbers from the set $\{1,2,3,4,5,6,7,8,9,10\}$. What is the smallest value that $\frac{a}{b}+\frac{c}{d}$ can take?

(**A**) $\frac{14}{45}$ (**B**) $\frac{1}{5}$ (**C**) $\frac{29}{90}$ (**D**) $\frac{3}{19}$ (**E**) $\frac{25}{72}$

B3 A 3×2 rectangle can be formed from two L-tiles  , for which there are exactly two possibilities:  and  . How many possibilities are there to form the figure shown on the right from four such L-tiles?

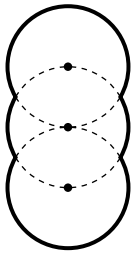


(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 8

B4 A triathlon consists of swimming, cycling and running. In our spring triathlon, three quarters of the total distance had to be covered by bike. The running distance was one fifth of the total distance. And the extra long swimming distance was 2 km long. What was the total distance of this triathlon?

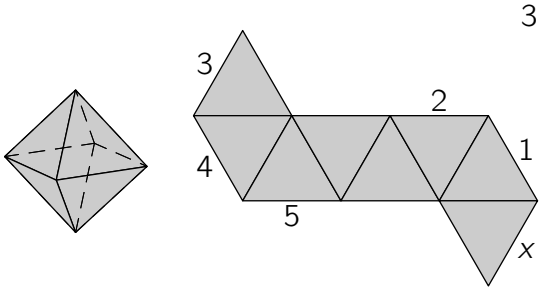
(**A**) 10 km (**B**) 20 km (**C**) 38 km (**D**) 40 km (**E**) 60 km

B5 The diagram shown is formed by three circles of equal size with a circumference of u. Their centres are on a straight line and on one or two of the other two circles. What is the length of the outer boundary of the figure?



(**A**) $\frac{3}{2}u$ (**B**) $\frac{4}{3}u$ (**C**) $\frac{5}{3}u$ (**D**) $2u$ (**E**) $\frac{5}{2}u$

B6 The octahedron shown was folded from the net shown next to it. Which edge in the net of the octahedron was glued to the edge labelled x?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

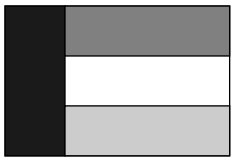
B7 A beverage is obtained by diluting a juice concentrate with water at the ratio 1 : 7. The juice concentrate is contained in a 1-litre bottle which is exactly half-full. What part of it should be taken to get 2 litres of this beverage?

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{2}{7}$ (**D**) $\frac{4}{7}$ (**E**) the whole quantity

B8 Ever since Ingrid, Petra and Renate retired, they have been going for a walk together once a day. If Ingrid doesn't wear a hat, then Petra wears one. If Petra doesn't wear a hat, then Renate wears one. Today Petra is without a hat. So, who wears a hat today?

(**A**) Ingrid and Renate (**B**) only Ingrid (**C**) only Renate
(**D**) neither Ingrid nor Renate (**E**) This cannot be said for sure.

B9 The flag of our fishing club must be sewn again. It is rectangular, and its height and its width are at a ratio of 3 : 5. The four differently coloured pieces of fabric are rectangular and have the same area (*diagram not to scale*). What is the ratio of the shorter to the longer side of the black rectangle?



(**A**) 1 : 3 (**B**) 2 : 5 (**C**) 2 : 7 (**D**) 3 : 10 (**E**) 5 : 12

B10 My seven-digit phone number $AAABBBB$ starts with three equal digits A and ends with four equal digits B. The sum of all seven digits is the two-digit number AB. Then $A + B =$

(**A**) 3 (**B**) 7 (**C**) 10 (**D**) 14 (**E**) 17

5 point problems

C1 Joel and his grandfather are at the market to buy fruit. "Grandpa, look what's on sale here," Joel shouts. "But read the price tags!" his grandfather grumbles. "How are we supposed to compare the prices?" "Well," Joel replies, "then we have to do the maths." Which of the following price tags belongs to the fruit that is the cheapest in relation to its weight?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

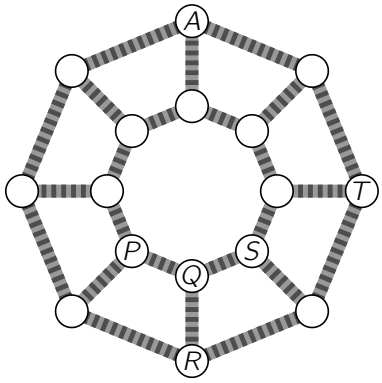
C2 There are 60 silver and 60 golden pearls to be divided into bowls in such a way that each bowl contains the same number of silver pearls but no two bowls contain the same number of golden pearls. What is the maximum number of bowls that can be filled this way?

(**A**) 20 (**B**) 15 (**C**) 12 (**D**) 10 (**E**) 6

C3 We imagine the numbers from 1 to 99 written in a row one after the other without gaps 1234567891011 . . . 9899. Now, three consecutive digits are combined into groups of three by parentheses (123)(456)(789)(101)(112) . . . (979)(899). Which of the following groups of three is not one of them?

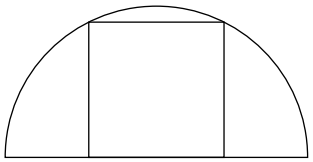
(**A**) (222) (**B**) (444) (**C**) (464) (**D**) (646) (**E**) (888)

C4 In the kindergarten there is the balancing device shown, where the children can balance over the connecting bars. To go from one corner to a neighbouring corner is called a step. Which of the corners P, Q, R, S, T can a child starting in A reach after 33 steps?



(**A**) only T (**B**) only Q
(**C**) only P, R and S (**D**) only P, R, S and T
(**E**) each of the 5 corners

C5 A square is inscribed in a semi-circle with a radius of 1 cm, i. e. two corners lie on the semi-circle and the other two on the diameter (see diagram). What is the area of the square?



(**A**) $\frac{4}{5}\,\text{cm}^2$ (**B**) $\frac{\sqrt{5}}{4}\,\text{cm}^2$ (**C**) $1\,\text{cm}^2$ (**D**) $\frac{4}{3}\,\text{cm}^2$ (**E**) $\frac{2}{\sqrt{3}}\,\text{cm}^2$

C6 How many solutions (x, y), where x and y are positive integers, does the equation $3x + 5y = 2019$ have?

(**A**) 15 (**B**) 118 (**C**) 119 (**D**) 134 (**E**) 135

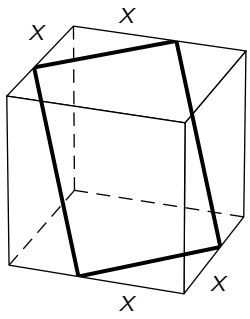
C7 With her calculator Florentine wants to calculate $(a + b) : c$, where a, b and c are natural numbers. She enters $\boxed{a + b \div c =}$ and gets the result 11. On the other hand, when entering $\boxed{b + a \div c =}$ she gets the result 14. What is the correct result of $(a + b) : c$ that Florentine is looking for?

(**A**) 1 (**B**) 2 (**C**) 5 (**D**) 7 (**E**) 8

C8 How many planes are there that contain exactly three corners of a given cube?

(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 12

C9 In the diagram, you can see a rectangle whose corners lie on the edges of a cube with side length 1. For what value of x is the rectangle a square?



(**A**) $\frac{1}{\sqrt{2}}$ (**B**) $\frac{2}{3}$ (**C**) $\frac{5}{4\sqrt{3}}$ (**D**) $\frac{\sqrt{3}}{2}$ (**E**) $\frac{3}{4}$

C10 A positive integer is written at each corner of a square. If two of the numbers belong to the same square side, one is a multiple of the other – and – of each two diagonally opposite numbers, none is a multiple of the other. What is the smallest possible sum of the four numbers?

(**A**) 24 (**B**) 31 (**C**) 35 (**D**) 47 (**E**) 53