Year 11 to 13 (ENGLISH VERSION)

Thursday, 17th March 2022 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

A1 $2^0 \times 2^2 =$

(**A**) 1 (**B**) 2 (**C**) 4 (**D**) 6 (**E**) 8

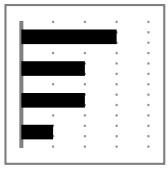
A2 At the end of a card game, Kai has more points than Zoé but less than Jona, and Svea has more points than Kai. Two of them have exactly the same number of points. Which two are they?

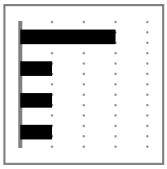
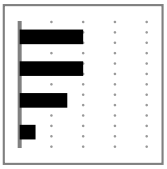
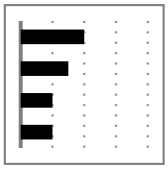
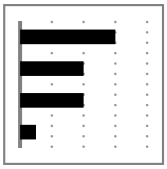
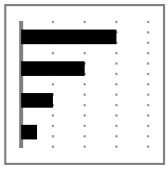
(**A**) Svea and Jona (**B**) Zoé and Svea (**C**) Jona and Zoé (**D**) Svea and Kai (**E**) Kai and Jona

A3 The product of the digits of a 10-digit natural number is 15. What is the sum of the digits of this number?

(**A**) 8 (**B**) 12 (**C**) 15 (**D**) 16 (**E**) 20

A4 On Maren's smartphone a diagram shows how much time she spent yesterday with her four favourite apps (see figure). Today, she halved the time spent on two of these apps, and she spent the same amount of time on the other two apps. In the diagram, the apps are always sorted by how long they were used. What can the diagram for today certainly not look like?



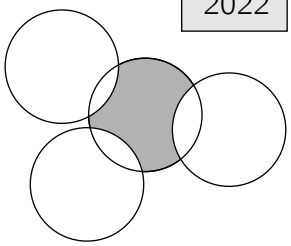
(**A**) 
(**B**) 
(**C**) 
(**D**) 
(**E**) 

A5 On a piece of paper, all the natural numbers from 2 to 2022 that consist only of the digits 0 and 2 are listed in ascending order. Which number is in the middle?

(**A**) 200 (**B**) 220 (**C**) 222 (**D**) 2000 (**E**) 2002

2
20
22
⋮
2022

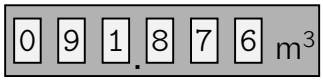
A6 The four circles shown all have radius 1 cm. What is true (in cm) for the perimeter u of the grey area?



(**A**) $u \leq \pi$ (**B**) $u = \dfrac{3\pi}{2}$ (**C**) $\dfrac{3\pi}{2} < u < 2\pi$

(**D**) $u = 2\pi$ (**E**) $u \geq 3\pi$

A7 Nick notices that there are six different digits on the water meter (see figure). How much water will be used until there are six different digits on the water meter again for the next time?

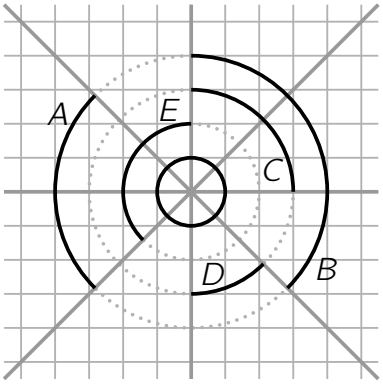


(**A**) 0.047 m^3 (**B**) 0.186 m^3 (**C**) 0.258 m^3 (**D**) 0.537 m^3 (**E**) 2.249 m^3

A8 For how many real numbers x is the equation $(x-2)^2+(x+2)^2=0$ true?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

A9 Which of the black arcs has the same length as the circumference of the small black circle in the middle?



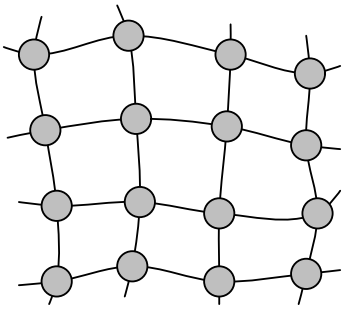
(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

A10 Let a, b, c be real numbers all of which are not equal to 0. The numbers $-a^4b^3c^2$ and $a^3b^5c^6$ have the same sign.
Which of the following statements is definitely true?

(**A**) $a>0$ (**B**) $b<0$ (**C**) $c>0$ (**D**) $b>0$ (**E**) $a<0$

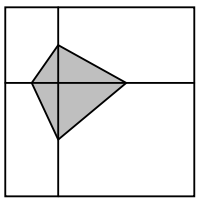
4 point problems

B1 In the city park, water dispensers are to be installed for joggers at some of the 16 intersections (see figure). What is the smallest number of water dispensers needed to ensure that from each intersection the nearest water dispenser is no more than one path segment away?



(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

B2 A large square is divided into 4 rectangles (see figure). The vertices of the grey quadrilateral are the midpoints of the sides of these rectangles. The area of the grey quadrilateral is $3\,\text{cm}^2$. What is the area of the large square?

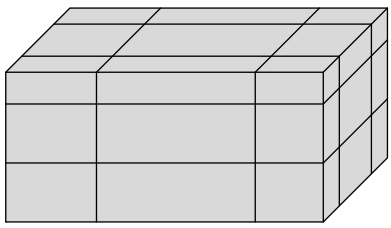


(**A**) $16\,\text{cm}^2$ (**B**) $18\,\text{cm}^2$ (**C**) $20\,\text{cm}^2$ (**D**) $24\,\text{cm}^2$ (**E**) $27\,\text{cm}^2$

B3 What is the greatest common divisor of $2^{2021}+2^{2022}$ and $3^{2021}+3^{2022}$?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 12 (**E**) 36

B4 The surface area of the cuboid shown is A. With six straight cuts it is divided into 27 smaller cuboids. What is the total surface area of all 27 small cuboids?



(**A**) $\frac{5}{2}A$ (**B**) $3A$ (**C**) $\frac{7}{2}A$ (**D**) $4A$ (**E**) $\frac{9}{2}A$

B5 If the weather forecast announced rain or if it rained the day before, Mrs Sugar would take her umbrella to work, otherwise not. Last week, she took her umbrella with her on Tuesday, Wednesday and Friday. On Monday and Thursday, she left it at home. The weather forecast was right on each of the 5 days. On how many of the 5 days did it rain?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

B6 On a straight line the points A, B, C and D were marked: 

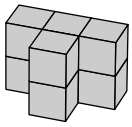
The distance between A and C is 12 cm and the distance between B and D is 18 cm. What is the distance between the midpoint of $\overline{AB}$ and the midpoint of $\overline{CD}$?

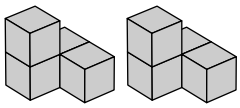
(**A**) 6 cm (**B**) 9 cm (**C**) 12 cm (**D**) 15 cm (**E**) 18 cm

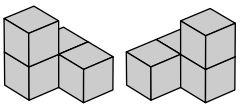
B7 The average (i.e. the arithmetic mean) of five numbers is 24, the average of the three smallest of these numbers is 19, and the average of the three largest is 28. What is the third largest of these numbers?

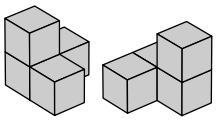
(**A**) 20 (**B**) 21 (**C**) 22 (**D**) 23 (**E**) 24

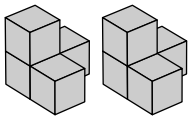
B8 Which two pieces can be put together to build the solid shown on the right?

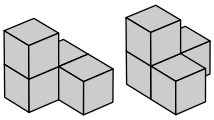


(**A**) 

(**B**) 

(**C**) 

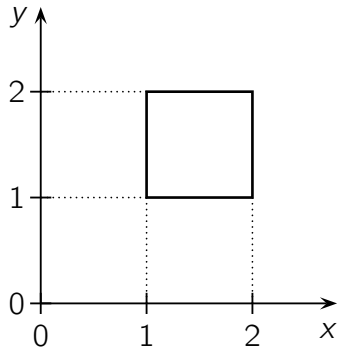
(**D**) 

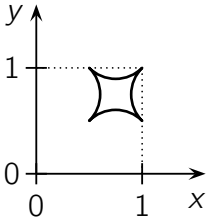
(**E**) 

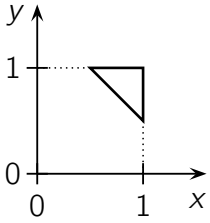
B9 There are 8 students taking part in a judo competition. The four pairings of the 1st round are drawn and the four winners will advance to the 2nd round. Then, the two pairings of the 2nd round will be drawn again and the two winners will advance to the final. Stephan will win every fight except the one against the invincible Frank, who will win the competition. What is the probability that Stephan will reach the final?

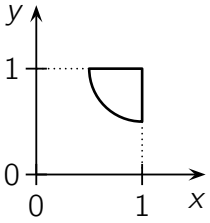
(**A**) $\frac{3}{7}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{4}{7}$ (**D**) $\frac{2}{3}$ (**E**) $\frac{7}{8}$

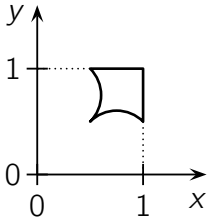
B10 In a coordinate system a square is drawn, as shown in the diagram on the right. Each point $(x\,|\,y)$ of the square is transformed to the point $\left(\frac{1}{x}\,\middle|\,\frac{1}{y}\right)$. What will the transformed square look like?

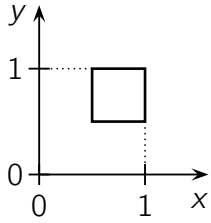


(**A**)


(**B**)


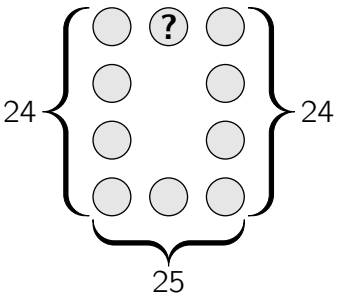
(**C**)


(**D**)


(**E**)


5 point problems

C1 Jenny wants to write the numbers from 1 to 10 in the circles in such a way, that the sum of the four numbers in the circles on the left and the sum of the four numbers in the circles on the right are both equal to 24 and the sum of the three numbers in the circles on the bottom is equal to 25. Which number must she write in the circle with the question mark?

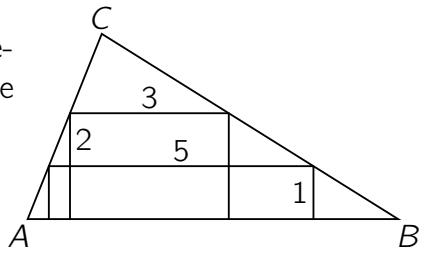


(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 6 (**E**) 8

C2 Malte writes the numbers from 1 to 20 at the vertices of a 20-sided polygon. Then, on each edge he writes the absolute value of the difference between the two numbers at the vertices of this edge. On each edge there is now either a 1 or a 2. On how many edges is there a 1?

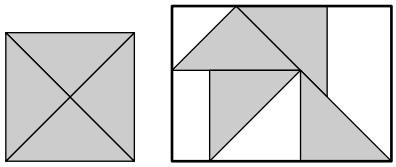
(**A**) 2 (**B**) 4 (**C**) 10 (**D**) 12 (**E**) 16

C3 A rectangle with side-lengths 1 cm and 5 cm and a rectangle with side-lengths 2 cm and 3 cm fit into the triangle ABC as shown. What is the height of this triangle with base $\overline{AB}$?



(**A**) $\frac{10}{3}$ cm (**B**) $\frac{7}{2}$ cm (**C**) $\frac{13}{4}$ cm (**D**) $\frac{16}{5}$ cm (**E**) $\frac{11}{3}$ cm

C4 Jasira cuts a colorful square sheet of paper with side-length 10 cm into four congruent triangles. She glues the four parts on the lid of a rectangular box (see figure). What is the area (in cm^2) of the part of the lid that Jasira did not cover?

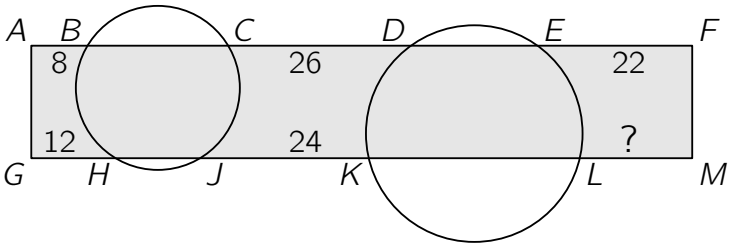


(**A**) $50+50\sqrt{2}$ (**B**) $125\sqrt{2}-100$ (**C**) $75+25\sqrt{2}$
(**D**) $250-100\sqrt{2}$ (**E**) $75\sqrt{2}$

C5 How many 3-digit natural numbers are there that are equal to 5 times the product of their digits?

(**A**) 5 (**B**) 4 (**C**) 3 (**D**) 2 (**E**) 1

C6 Two circles intersect the rectangle $GMFA$, as shown. It is given that $|AB| = 8$ cm, $|CD| = 26$ cm, $|EF| = 22$ cm, $|GH| = 12$ cm and $|JK| = 24$ cm. What is the distance between the points L and M?

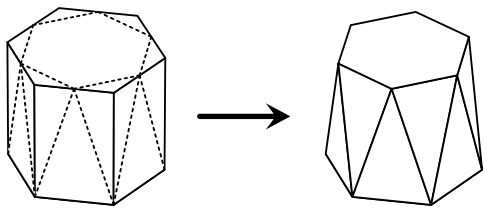


(**A**) 14 cm (**B**) 15 cm (**C**) 16 cm
(**D**) 17 cm (**E**) 18 cm

C7 Let N be an arbitrary positive integer. How many integers are greater than $\sqrt{N^2+N+1}$ and less than $\sqrt{9N^2+N+1}$?

(**A**) $N+1$ (**B**) $2N-1$ (**C**) $2N$ (**D**) $2N+1$ (**E**) $3N-1$

C8 From a regular 6-sided prism the six upper corners were cut off (see figure). The new top surface is now a smaller regular hexagon and the lateral surface consists of twelve isosceles triangles. The volume of the prism was 36 cm^3. What is the total volume of the six corners that were cut off?



(**A**) 3 cm^3 (**B**) $3\sqrt{3}$ cm^3 (**C**) $3\sqrt{2}$ cm^3 (**D**) 6 cm^3 (**E**) $2\sqrt{3}$ cm^3

C9 A sequence f_n is defined by $f_{2m} = f_2 \times f_m + 1$ and $f_{2m+1} = f_2 \times f_m - 2$ for all $m \geq 1$. Of the first element we know that $0 < f_1 < 1$, and we also know that $f_7 = 2$. What is the value of f_2?

(**A**) 2 (**B**) $\frac{7}{2}$ (**C**) -1 (**D**) $-\frac{1}{3}$ (**E**) 4

C10 At a jeweller, the value of the diamonds in the safe is 45 times the value of the diamonds in the display. When a customer reserved the two most valuable diamonds from the display, the jeweller took them to the safe to keep them there for him. Now the value of the diamonds in the safe is 48 times the value of the diamonds in the display. What is the smallest possible number of diamonds that are now left in the display?

(**A**) 19 (**B**) 24 (**C**) 27 (**D**) 31 (**E**) 36